Solar System Experiments and the Interpretation of Saa's Model of Gravity with Propagating Torsion as a Theory with Variable Plank "Constant"

P. Fiziev [*] [†]

Bogoliubov Laboratory of Theoretical Physics

Joint Institute for Nuclear Research 141980 Dubna, Moscow Region, Russia

S. Yazadjiev [‡]

Department of Theoretical Physics, Faculty of Physics, Sofia University,

5 James Bourchier Boulevard, Sofia 1164, Bulgaria

February 7, 2008

Abstract

It is shown that the recently proposed interpretation of the transposed equi-affine theory of gravity as a theory with variable Plank "constant" is inconsistent with basic solar system gravitational experiments.

Recently a new model of gravity involving propagating torsion was proposed by A. Saa [1]-[5]. In this model a special type of Einstein-Cartan geometry is considered in which the usual volume element $\sqrt{-g}\,d^4x$ is replaced with new one: $e^{-3\Theta}\sqrt{-g}\,d^4x$ – covariantly constant with respect to the transposed affine connection ∇^T, hence the name transposed-equi-affine theory of gravity [6]. As a result the torsion vector $S_\alpha = S_{\alpha\beta}{}^{\alpha}$ turns to be potential: $S_\alpha = \partial_\alpha\Theta$, Θ being its scalar potential [1].

Because of the exponential factor $e^{-3\Theta}$ in the volume element Saa's model has a very important feature: it leads to a consistent application of the minimal

[*]E-mail: fiziev@phys.uni-sofia.bg

[†]Permanent address: Department of Theoretical Physics, Faculty of Physics, Sofia University, 5 James Bourchier Boulevard, Sofia 1164, Bulgaria

[‡]E-mail: yazad@phys.uni-sofia.bg

[1]We use the Schouten's normalization conventions [7] which differs from the original ones in [1]–[5].

coupling principle both in the action principle and in the equations of motion for all matter fields. These equations are of autoparallel type and may be derived via the standard action principle for *a nonstandard action integral*:

$$\mathcal{A}_{tot} = \mathcal{A}_G + \mathcal{A}_{MF} = \frac{1}{c}\int \mathcal{L}_G\, e^{-3\Theta}\sqrt{-g}d^4x + \frac{1}{c}\int \mathcal{L}_{MF}\, e^{-3\Theta}\sqrt{-g}d^4x, \qquad (1)$$

where $\mathcal{L}_G = -\frac{c^2}{2\kappa}R$ is the lagrangian of the geometrical fields: the metric $g_{\alpha\beta}$, and the torsion $S_{\alpha\beta}{}^{\gamma}$, R being the Cartan scalar curvature, c being the speed of light, and $\mathcal{L}_{MF}$ is the usual lagrangian of the corresponding matter fields: scalar fields $\phi(x)$, spinor fields $\psi(x)$, electromagnetic fields $A_\alpha(x)$, Yang-Mills fields $\mathbf{A}_\alpha(x)$, e.t.c.

But this property not held for the equations of motion of classical particles and fluids which turn to be of geodesic type [6]. Most probably this inconsistency leads to the negative result obtained in [8]: Saa's model is inconsistent with the basic solar system gravitational experiments.

Then having in mind to preserve the good features of Saa's model and in the same time somehow to avoid this problem we are forced to try some further modifications of the model. The simplest one is to use the Saa's modification of the volume element *only in the action integrals* like (1) and the usual volume element in all other physical, or geometrical formulae [6]. This leads to the action for a classical spinles particle in a form:

$$\mathcal{A}_m = -mc\int e^{-3\Theta}ds \qquad (2)$$

where m is the rest mass of the particle and $ds = \sqrt{g_{\alpha\beta}dx^\alpha dx^\beta}$ is the usual four-dimensional interval. The corresponding action integral for spinless fluid (See for details [6]) is:

$$\mathcal{A}_\mu = \frac{1}{c}\int \mathcal{L}_\mu\, e^{-3\Theta}\sqrt{-g}d^4x = -\frac{1}{c}\int (\mu c^2 + \mu\Pi)\, e^{-3\Theta}\sqrt{-g}d^4x, \qquad (3)$$

$\mu(x)$ being fluid's density, Π being the elastic potential energy of the fluid.

This situation calls for a new curious interpretation of the torsion potential Θ as a quantity which describes the space-time variations of the Plank "constant" according to the law

$$\hbar(x) = \hbar_\infty e^{3\Theta(x)}, \qquad (4)$$

$\hbar_\infty$ being the Plank constant in vacuum far from matter.

Indeed, according to the first principles we actually need lagrangians and action integrals to write down the quantum transition amplitude in a form of Feynman path integral on the histories of all fields and particles. In the variant of the theory under consideration it has the form:

$$\int \mathcal{D}\left(g_{\alpha\beta}(x), S_{\alpha\beta}{}^{\gamma}(x), \phi(x), \psi(x), A_\alpha(x), \mathbf{A}_\alpha(x), ...; x(t), ...\right)$$
$$\exp\left(\frac{1}{\hbar_\infty}\left(\int d^4x e^{-3\Theta(x)}(L_G + L_{MF}) - mc\int e^{-3\Theta}ds\right)\right). \qquad (5)$$

Now it is obvious that the very Plank constant $\hbar$ may be included in the factor $e^{3\Theta(x)}$, but more important is the observation that we must do this, because the presence of this *uniform* factor in the formula (5) means that we actually introduce a local Plank "constant" at each point of the space-time. Indeed, if the geometric field $\Theta(x)$ changes slowly in a cosmic scales, then in the framework of the small domain of the laboratory we will see an effective "constant": $\hbar(x) \approx \hbar_\infty e^{3\Theta(x_{laboratory})} = const = \hbar$.

In presence of spinless matter only an Einstein-Cartan geometry with semi-symmetric torsion tensor $S_{\alpha\beta}{}^{\gamma} = S_{[\alpha}\delta^{\gamma}_{\beta]}$ appears and the following equations for geometrical fields are obtained

$$\begin{aligned} G_{\mu\nu} + \nabla_\mu\nabla_\nu\Theta - g_{\mu\nu}\Box\Theta &= \frac{\kappa}{c^2}\left((\varepsilon+p)u_\mu u_\nu - pg_{\mu\nu}\right) \\ \nabla_\sigma S^\sigma = \Box\Theta &= -\frac{2\kappa}{c^2}\left(\varepsilon+3p\right) \end{aligned} \tag{6}$$

where $G_{\mu\nu}$ is Einstein tensor with respect to the affine connection ∇_μ, κ being the Einstein gravitational constant, ε, p and u_μ are the energy density, pressure and four velocity of the relativistic perfect fluid [6]. Using the standard variational principle for the action (3) one can obtain the equations of motion for the perfect fluid:

$$(\varepsilon+p)u^\beta\nabla_\beta u_\alpha = \left(\delta^\beta_\alpha - u_\alpha u^\beta\right)\nabla_\beta p + \mathcal{F}_\alpha \tag{7}$$

where

$$\mathcal{F}_\alpha = -2(\varepsilon+p)\left(\delta^\beta_\alpha - u_\alpha u^\beta\right)\nabla_\beta\Theta$$

is the torsion force, as defined in [6].

This nonzero value of the torsion force shows that in the present model with variable Plank "constant" (VPC model) the matter equations of motion are not of autoparallel, nor of geodesic type in contrast to all equations for matter fields which are of autoparallel type. This inconsistency of the model is not enough to reject it immediately as far as the very requirement for all dynamical equations in theory to be of the same type is not founded on a well established principle, nevertheless it seems to be necessary for validity of the corresponding generalization of the equivalence principle in spaces with torsion [9].

The main purpose of this letter is to investigate the consistency of the VPC model with basic solar-system experimental facts. To do this we have to consider the motion of a test particle in presence of a metric and torsion fields. The standard variation of the action (2) yields the equations of motion we need, but it's more convenient to investigate directly the corresponding Hamilton-Jacoby equation:

$$g^{\mu\nu}\partial_\mu S\partial_\nu S = \left(mce^{-3\Theta}\right)^2. \tag{8}$$

The conform transformation $g_{\mu\nu} \to \overset{*}{g}_{\mu\nu} = e^{-6\Theta}g_{\mu\nu}$ yields the effective metric $\overset{*}{g}_{\mu\nu}$ and the following form of the equation (8)

$$\overset{*}{g}{}^{\mu\nu}\partial_\mu S\partial_\nu S = m^2c^2 \tag{9}$$

which is well known from general relativity. Thus we may consider the motion of a test particles precisely as in general relativity working with the metric $\overset{*}{g}_{\mu\nu}$. Therefore the simplest way to compare the predictions of the VPC model with the experimental facts is to consider post-Newtonian expansion of the metric $\overset{*}{g}_{\mu\nu}$ in vacuum in vicinity of a star like the Sun.

The asymptotically flat, static and spherically symmetric general solution of the equations (6) for geometric fields in vacuum is known [11], [12]. In isotropic coordinates it's given by a two-parameter – (r_0, k) family of solutions

$$ds^2 = \left(\frac{1-\frac{r_0}{r}}{1+\frac{r_0}{r}}\right)^{\frac{2}{\rho(k)}} (c\,dt)^2 - \left(1-\frac{r_0^2}{r^2}\right)^2 \left(\frac{1-\frac{r_0}{r}}{1+\frac{r_0}{r}}\right)^{\frac{2}{\rho(k)}(3k-1)} \left(dr^2 + r^2 d\Omega^2\right), \quad (10)$$

$$\Theta = \frac{k}{2}\nu \quad (11)$$

where $\rho(k) = \sqrt{3\left(k-\frac{1}{2}\right)^2 + \frac{1}{4}}$. In the VPC model under consideration the whole geometry (metric and torsion) causes a gravitational force (of pure geometrical nature). The parameter k presents the ratio of the torsion part of this force and its metric part. In the case when $k = 0$ we have the usual torsionless Schwarzshild's solution and $r_g \equiv 4r_0$ is the standard gravitational radius.

From equations (11) we obtain the effective metric $\overset{*}{g}_{\mu\nu}$ and the effective four-interval

$$d\overset{*}{s}{}^2 = \left(\frac{1-\frac{r_0}{r}}{1+\frac{r_0}{r}}\right)^{\frac{2}{\rho(k)}(1-3k)} (c\,dt)^2 - \left(1-\frac{r_0^2}{r^2}\right)^2 \left(\frac{1-\frac{r_0}{r}}{1+\frac{r_0}{r}}\right)^{\frac{-2}{\rho(k)}} \left(dr^2 + r^2 d\Omega^2\right). \quad (12)$$

The asymptotic expansion of the metric in (12) at $r \to \infty$ gives

$$d\overset{*}{s}{}^2 \approx \left(1 - \frac{4r_0(1-3k)}{\rho(k)r} + \frac{8r_0^2(1-3k)^2}{\rho(k)^2 r^2}\right)(c\,dt)^2 - \left(1+\frac{4r_0}{\rho(k)r}\right)\left(dr^2 + r^2 d\Omega^2\right). (13)$$

In the asymptotic region $r \to \infty$ we must have Newtonian gravity. Consequently the mass "seen" by the test particles is

$$M = \frac{2r_0(1-3k)}{\rho(k)}. \quad (14)$$

Therefore we may represent the effective four-interval in the asymptotic form

$$d\overset{*}{s}{}^2 \approx \left(1 - \frac{2M}{r} + \frac{2M^2}{r}\right)(c\,dt)^2 - \left(1 + \frac{1}{1-3k}\frac{2M}{r}\right)\left(dr^2 + r^2 d\Omega^2\right). \quad (15)$$

From the above expression it immediately follows that two of post-Newtonian parameters corresponding to the effective metric $\overset{*}{g}_{\mu\nu}$ are

$$\overset{*}{\beta} = 1, \quad \overset{*}{\gamma} = \frac{1}{1-3k}. \quad (16)$$

As it's well known, solar system gravitational experiments set tight constrains on post-Newtonian parameters [14]:

$$|\overset{*}{\beta} -1 |< 1 * 10^{-3}, \quad |\overset{*}{\gamma} -1 |< 2 * 10^{-3}. \tag{17}$$

Therefore, to avoid contradictions with the basic experimental facts we must have

$$\left|\frac{3k}{1-3k}\right| < 2 * 10^{-3}. \tag{18}$$

In order to specify the theoretically possible values of k we must investigate a model of a star. As a simplest basic model we may consider a static spherically symmetric star. Putting the metric in the standard form

$$ds^2 = e^{\nu}(c\,dt)^2 - e^{\lambda}dr^2 - r^2(d\theta^2 + \sin^2(\theta)d\varphi^2)$$

we obtain from the general field equations (6), (7) the following complete system of ordinary differential equations for the star's fluid equilibrium

$$\begin{aligned}
\xi' + \frac{2}{r}\xi + \left(\frac{2\xi - \lambda'}{2}\right)\xi - 3S_r\xi &= -(\varepsilon + 3p)e^{\lambda} \\
S_r' + \frac{2}{r}S_r + \left(\frac{2\xi - \lambda'}{2}\right)S_r - 3S_r^2 &= -(\varepsilon + 3p)e^{\lambda} \\
e^{\lambda} - \left(1 + \frac{r}{2}(2\xi - \lambda')\right) &= -3rS_r + 2(\varepsilon + 2p)r^2e^{\lambda} \\
\xi' - \frac{\lambda'}{2}\xi + \xi^2 - \frac{\lambda'}{r} &= 3S_r' - \frac{3}{2}\lambda' S_r + 3S_r^2 - 2(\varepsilon + 2p)e^{\lambda} \\
p' &= -(\varepsilon + p)(\xi - 3S_r) \\
p &= p(\varepsilon)
\end{aligned} \tag{19}$$

where $\xi = \frac{1}{2}\nu'$, $S_r = \Theta'$, $p = p(\varepsilon)$ is the matter state equation, ε and p are the energy density and the pressure. The prime denotes differentiation with respect to r.

The regular at the center of the star ($r = 0$) solution corresponds to the initial conditions [13]:

$$\xi(0) = 0, \;\; S_r(0) = 0.$$

As we see the first two equations of the system (19) coincide. Then by virtue of the same initial conditions for ξ and S_r we obtain equal solutions $\xi = S_r$. Hence, in VPC model the only possible value of the parameter k is $k = 1$. This means that in this model the torsion part of gravitational force equals to the metric one in magnitude. As a consequence it is impossible to fulfill the condition (18). Moreover, if $r_0 > 0$ the value $k = 1$ leads to a negative mass of the star (See equation (14)).

This result shows that the interpretation of the Saa's model as a theory with variable Plank's constant is inconsistent with the well known solar system gravitational experiments.

Acknowledgments

This work has been partially supported by the Sofia University Foundation for Scientific Researches, Contract No. 245/98, and by the Bulgarian National Foundation for Scientific Researches, Contract F610/98.

One of us (PF) is grateful to the leadership of the Bogoliubov Laboratory of Theoretical Physics, JINR, Dubna, Russia for hospitality and working conditions during his stay there in the summer of 1998.

References

[1] A. Saa, Gen. Rel. and Grav. **29**, 205, (1997).

[2] A. Saa, Mod. Phys. Lett. **A8**, 2565, (1993).

[3] A. Saa, Mod. Phys. Lett. **A9**, 971, (1994).

[4] A. Saa, Class. Quant. Grav. **12**, L85, (1995).

[5] A. Saa, J. Geom. and Phys. **15**, 102, (1995).

[6] P. Fiziev, *Spinless matter in Transposed-Equi-Affine theory of gravity*, E-print gr-qc/9712004, to appear in the journal General Relativity and Gravitation.

[7] J. A. Schouten, *Tensor Analysis for Physicists*, (Clarendon Press, Oxford, 1954); *Ricci-Calculus* (Springer, Berlin, 1954).

[8] P. Fiziev, S. Yazadjiev, *Solar-system experiments and Saa's Model of gravity with Propagating Torsion*, E-print gr-qc/9806063.

[9] P. Fiziev, *On the Action Principle in Affine Flat Spaces with Torsion*, in the Proceedings of the 2-nd Bulgarian Workshop "New Trends in Quantum Field Theory", Razlog, 1995, editors A. Ganchev, R.Kerner, I.Todorov, Heron Press Science Series, Sofia, 248, (1996); E-print: gr-qc/9712002.

[10] C. Brans, R. H. Dicke, Phys. Rev. **124**, 925 (1961).

[11] C. Brans, Phys. Rev. **125**, 2194 (1961).

[12] B. C. Xanthopoulos, T. Zannias Phys. Rev. **D40**, 2564 (1989).

[13] T. Boyadjiev, P. Fiziev, S. Yazadjiev *Neutron star in presence of torsion-dilaton field*, E-print gr-qc/9803084.

[14] C. M. Will, *Theory and Experiment in Gravitational Physics*, (Cambridge University Press, 1993).